

**Section A**

1.

**(d) Option C****Explanation:** ${}^6\text{C}$ :  $1\text{s}^2 2\text{s}^2 2\text{p}^2$  ${}^{14}\text{Si}$ :  $1\text{s}^2 2\text{s}^2 2\text{p}^6 3\text{s}^2 3\text{p}^2$ 

The energy required to take out an electron from the 3rd orbit of Si is much smaller than to take out an electron from the 2nd orbit of C. So, Si has a significant number of free electrons while C has a negligibly small number of free electrons.

2.

**(d)  $9.6 \times 10^{+3} \text{ A m}^{-2}$** **Explanation:**

To find the current density in the electron beam, we first need to calculate the current ( $I$ ) flowing through the beam. The current can be determined using the formula:  $I = n \cdot e$  where  $n$  is the number of electrons passing through per second per unit area, and  $e$  is the charge of an electron (approximately  $1.6 \times 10^{-19}$  coulombs).

Given that  $n = 6.0 \times 10^{16}$  electrons/m<sup>2</sup>/s, we can calculate the current for the given aperture area of  $1.0 \text{ mm}^2$  (which is  $1.0 \times 10^{-6} \text{ m}^2$ ):

i. Calculate the total number of electrons passing through the aperture per second:

$$\text{Total electrons} = n \cdot \text{Area} = 6.0 \times 10^{16} \text{ electrons/m}^2/\text{s} \times 1.0 \times 10^{-6} \text{ m}^2 = 6.0 \times 10^{10} \text{ electrons/s}$$

ii. Now, calculate the current:  $I = 6.0 \times 10^{10} \text{ electrons/s} \times 1.6 \times 10^{-19} \text{ C/electron} = 9.6 \times 10^{-9} \text{ A}$

iii. Finally, to find the current density  $J$ , we use the formula:  $J = \frac{I}{\text{Area}} = \frac{9.6 \times 10^{-9} \text{ A}}{1.0 \times 10^{-6} \text{ m}^2} = 9.6 \times 10^3 \text{ A/m}^2$

This calculation confirms that the current density in the beam is  $9.6 \times 10^3 \text{ A/m}^2$ , which matches option  $(9.6 \times 10^{+3} \text{ A m}^{-2})$ .

3.

**(b)  $\frac{R}{2}$** **Explanation:**

The relationship between **the focal length f** and **radius of curvature r** for spherical mirror is given by  $R = 2f$ . Therefore,

$$f = \frac{R}{2}$$

4.

**(b) 0 and 0****Explanation:**

$$\text{Torque} = \tau = MB\sin \theta$$

Since, M and B are parallel, then  $\theta = 0$  and hence,

$$\tau = 0$$

Torque is 0. So, in this case force is also zero since the distance is not equal to zero.

5.

**(b) 2 C****Explanation:**

The equivalent capacitance between points A and B is calculated based on the configuration of the capacitors in the circuit. If the capacitors are in series, the formula for equivalent capacitance is given by:  $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$  If they are in parallel, the formula is:  $C_{eq} = C_1 + C_2 + \dots$  The solution indicates that the equivalent capacitance is 2 C, which suggests that the combination of capacitors results in this value based on the above formulas. The specific arrangement and values of the capacitors lead to this final result.

6. (a) perpendicular to the magnetic field

**Explanation:**

$$F = B I \sin \theta$$

$\theta$  is the angle between the direction of current and the direction of magnetic field.

So, when  $\theta = 90^\circ$ , the force is maximum.

7.

(d) Zero

**Explanation:**

Induced EMF is zero because flux linked with it remains constant.

8.

(d) Absolute Permittivity

**Explanation:**

Absolute Permittivity

9.

(b) interference, in which width of the fringe will be slightly increased

**Explanation:**

Strictly speaking, the refractive index of air is  $1.00029$  and that of vacuum is 1. Therefore, on evacuating the chamber, the wavelength of the light used will increase slightly. Since  $\beta \propto \lambda$ , the fringe width will increase slightly.

10. (a) the inverse square law was not exactly true

**Explanation:**

Gauss's law is based on the inverse square dependence of distance contained in the Coulomb's law. Any violation of Gauss's law will indicate departure from the inverse square law.

11.

(d) C and A

**Explanation:**

In both figures (A) and (C), p-side is at higher potential than the n-side.

12.

(d) smaller

**Explanation:**

When light travels from air to a medium of refractive index  $\mu$ , its wavelength decreases by a factor  $\mu$  i.e. becomes  $1/\mu$ .

13.

(c) A is true but R is false.

**Explanation:**

A photocell works on the principle of photoelectric emission. It is also called an electric eye.

14.

(b) Both A and R are true but R is not the correct explanation of A.

**Explanation:**

Both A and R are true but R is not the correct explanation of A.

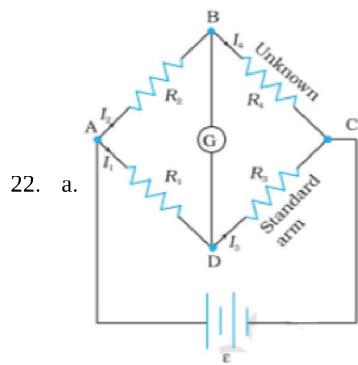
15.

(d) Both A and R are false.

**Explanation:**

When a light wave travel from a rarer to a denser medium it loses speed, but energy carried by the wave does not depend on its speed. Instead, it depends on the amplitude of wave. The frequency also remain constant.





Applying Kirchoff's loop rule to ADBA and CBDC

$$-I_1 R_1 + 0 + I_2 R_2 = 0$$

$$I_2 R_4 + 0 - I_1 R_3 = 0$$

Since,  $I_g = 0$ ,  $I_3 = I_1$ ,  $I_4 = I_2$

$$\frac{I_1}{I_2} = \frac{R_4}{R_3} \quad \text{and} \quad \frac{I_1}{I_2} = \frac{R_2}{R_1}$$

$$\therefore \frac{R_4}{R_3} = \frac{R_2}{R_1} \quad (\text{Balance Condition})$$

b. A practical device using the principle of Wheatstone bridge is meter bridge.

23. a.  $\lambda = 200 \text{ nm}$ , stopping potential =  $-2.5 \text{ V}$

$$\text{K.E.} = 2.5 \times 1.6 \times 10^{-19} \text{ J} = 4 \times 10^{-19} \text{ J}$$

$$\begin{aligned} E &= h\nu = h\frac{c}{\lambda} \\ &= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{200 \times 10^{-9}} \\ &= 9.945 \times 10^{-19} \text{ J} \end{aligned}$$

$$\text{Work function} = E - eV$$

$$= (9.945 \times 10^{-19} - 4 \times 10^{-19}) \text{ J}$$

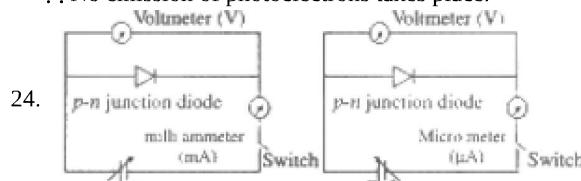
$$= 5.95 \times 10^{-19} \text{ J}$$

b. Wavelength of red light =  $6328 \text{ } \textcircled{\text{A}}$

$$\begin{aligned} E &= h\nu = h\frac{c}{\lambda} \\ &= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{6328 \times 10^{-10}} \\ &= 0.00314 \times 10^{-16} \text{ J} \\ &= 3.14 \times 10^{-19} \text{ J} \end{aligned}$$

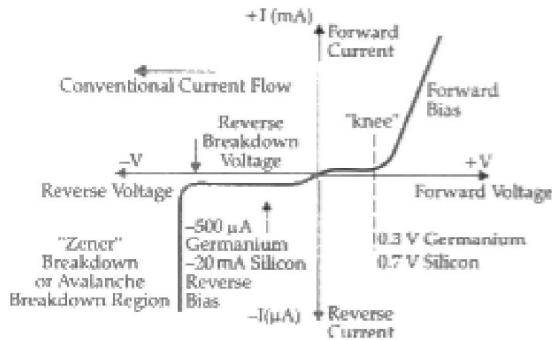
Energy of red light < work function of metal surface

$\therefore$  No emission of photoelectrons takes place.



In forward bias, the applied voltage does not support potential barriers. As a result, the depletion layer width decreases, and the barrier height is reduced. Due to the applied voltage, electrons from n side cross the depletion region and reach p side. Similarly, holes from p side cross the junction and reach the n side. The motion of charge carriers, on either side, give rise to current. In reverse bias, applied voltage supports potential barrier. As a result, the barrier height is increased, depletion layer widens. This suppresses the flow of electrons from n  $\rightarrow$  p and holes from p  $\rightarrow$  n, thereby decrease the diffusion current. The electric field direction of the junction is such that if electrons on p side or holes on n side in their random motion come close to the junction. They will be swept to its majority zone. This drift of carriers give rise to the current called reverse current. Also, Resistance of P-n

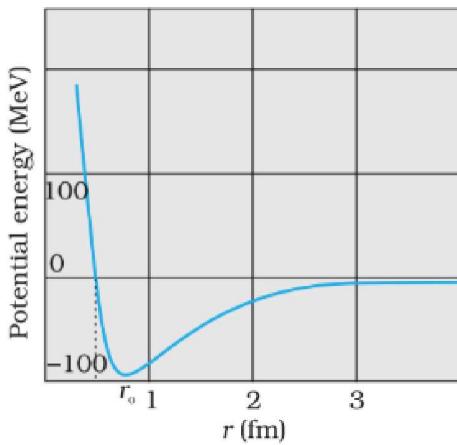
Junction in reverse bias is high.



25. i. The nuclear force binds nucleons into atomic nuclei. Characteristics properties of nuclear force are:

- a. Nuclear forces act between a pair of neutrons, a pair of protons and also between a neutron-proton pair, with the same strength. This shows that nuclear forces are independent of charge.
- b. The nuclear forces are dependent on spin or angular momentum of nuclei.
- c. Nuclear forces are non-central forces. This shows that the distribution of nucleons in a nucleus is not spherically symmetric.

ii. A plot of potential energy of a pair of nucleons as a function of their separation is shown below:



From the plot, it is concluded that

i. The potential energy is minimum at a distance  $r_0$  ( $\approx 0.8\text{ fm}$ ) which means that the force is attractive for distances larger than 0.8 fm and repulsive for the distance less than 0.8 fm between the nucleons

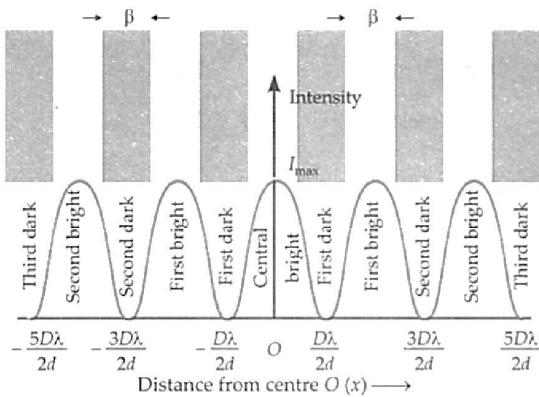
ii. Nuclear forces are negligible when the distances between the nucleons is more than 10 fm.

26. a. Given: The total energy of an electron in the first excited state of the hydrogen atom is about  $-3.4\text{ eV}$ .  
 The kinetic energy of the electron in this state = negative of the total energy =  $-E$   
 Kinetic energy of the electron in this state =  $-(-3.4)\text{eV} = + 3.4\text{ eV}$

b. Potential energy is given as the negative of the twice of the kinetic energy  $U = -2 \times (3.4)\text{ eV}$   
 $U = -6.8\text{eV}$   
 Hence the potential energy of the electron in the given state is  $- 6.8\text{ eV}$ .

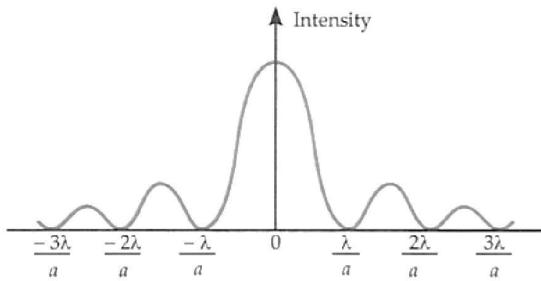
c. If the choice of the zero of potential energy is changed, then the value of potential energy of the system also changes and as we know the total energy is the sum of kinetic energy as well as potential energy. Therefore, the potential energy will also change.

27. With two narrow slits, an interference pattern is obtained.  
 When one slit is completely covered, the diffraction pattern is obtained.  
 For intensity distribution curve for interference, see Fig.



Intensity distribution curve.

For intensity distribution curve for diffraction, see Fig.



Interference	Diffraction
1. All the bright fringes are of same intensity.	Intensity of bright fringes decreases with the increasing order.
2. All the bright fringes are of equal width.	Central bright fringe is twice as wide as any secondary bright fringe.
3. Regions of dark fringes are perfectly dark.	Regions of dark fringes are not perfectly dark.
4. Maxima occur at $\theta = n\frac{\lambda}{d}$	Minima occur at $\theta = n\frac{\lambda}{a}$

28. i. Let RS moves with speed  $v$  rightward and also RS is at distances  $x_1$  and  $x_2$  from PQ at instants  $t_1$  and  $t_2$ , respectively.

Change in flux,  $d\phi = \phi_2 - \phi_1 = Bl(x_2 - x_1)$  [since magnetic flux,  $\phi = \vec{B} \cdot \vec{A} = BA\cos 0^\circ = Blx$ ]

$$\Rightarrow d\phi = Bl dx \Rightarrow \frac{d\phi}{dt} = Bl \frac{dx}{dt} = Blv \quad \left[ \because v = \frac{dx}{dt} \right]$$

If resistance of loop is  $R$ , then  $I = \frac{vBl}{R}$

ii. Magnetic force =  $BIl \sin 90^\circ$

$$= \left( \frac{vBl}{R} \right) Bl = \frac{vB^2 l^2}{R}$$

Now, External force must be equal to magnetic force

$$\therefore \text{External force} = \frac{vB^2 l^2}{R}$$

$$\text{iii. As, } P = I^2 R = \left( \frac{vBl}{R} \right)^2 \times R = \frac{v^2 B^2 l^2}{R^2} \times R$$

$$\therefore P = \frac{v^2 B^2 l^2}{R}$$

OR

Self-inductance of a coil is the property of the coil in which it opposes the change of current flowing through it. Inductance is attained by a coil due to the self-induced emf produced in the coil itself by changing the current flowing through it.

Self-induction of the long solenoid of inductance  $L$ , (A long solenoid is one which length is very large as compared to its cross-section area.) the magnetic field inside such a solenoid is constant at any point and given by

$$B = \frac{\mu_0 NI}{l}$$

Magnetic flux through each turn of solenoid

$$\phi = B \times \text{area of each turn}$$

$$\phi = \frac{\mu_0 NI}{l} \times A$$

total flux = flux  $\times$  total number of turns

$$N\phi = N \left( \frac{\mu_0 NI}{l} \times A \right) \dots (i)$$

If L is the coefficient of inductance of solenoid

$$N\phi = LI \dots \text{(ii)}$$

from equation (i) and (ii)

$$LI = N \left( \frac{\mu_0 NI}{l} \times A \right)$$

$$L = \frac{\mu_0 N^2 A}{l} \dots \text{(iii)}$$

The magnitude of emf is given by

$$|e| \text{ or } e = L \frac{dI}{dt} \dots \text{(iv)}$$

multiplying I to both sides

$$eIdt = LIdt$$

$$\text{but } I = \frac{dq}{dt}$$

$$Idt = dq$$

Also work done ( $dW$ ) = voltage  $\times$  Charge( $dq$ )

$$\text{or } dW = e \times dq = eIdt$$

substituting the values in equation (iv)

$$dW = LIdt$$

By integrating both sides

$$\int_0^w dW = \int_0^{I_0} LIdt$$

$$W = \frac{1}{2} LI_0^2$$

this work done is in increasing the current flow through inductor is stored as potential energy (U) in the magnetic field of inductor

$$U = \frac{1}{2} LI_0^2$$

## Section D

### 29. Read the text carefully and answer the questions:

In an electromagnetic wave both the electric and magnetic fields are perpendicular to the direction of propagation, that is why electromagnetic waves are transverse in nature. Electromagnetic waves carry energy as they travel through space and this energy is shared equally by the electric and magnetic fields. Energy density of an electromagnetic waves is the energy in unit volume of the space through which the wave travels.

(i) **(d)**  $\vec{E} \times \vec{B}$

**Explanation:**

Electromagnetic waves propagate in the direction of  $\vec{E} \times \vec{B}$ .

(ii) **(b)** photon

**Explanation:**

Photon is the fundamental particle in an electromagnetic wave.

(iii) **(a)** polarisation

**Explanation:**

Polarisation establishes the wave nature of electromagnetic waves.

OR

**(b)** in phase and perpendicular to each other

**Explanation:**

The electric and magnetic fields of an electromagnetic wave are in phase and perpendicular to each other.

(iv) **(d)** frequency

**Explanation:**

Frequency  $\nu$  remains unchanged when a wave propagates from one medium to another. Both wavelength and velocity get changed.

### 30. i. The total flux coming out is $\epsilon_0^{-1}$ .

$$\text{ii. } \phi = EA \cos \theta$$

$$= (2)(1)(\cos 90^\circ - 30^\circ)$$

$$= (2)(1)(\cos 60^\circ)$$

$$= (2)(1)(\frac{1}{2})$$

$$= 1 \text{ Vm}$$

iii. It will be positive when the flux lines are directed outwards.

iv. It depends on the net charge enclosed.

v. It is when the surface is perpendicular to the field.

### Section E

31. i. Assuming the aperture of the surface is small as compared to other distance involved, so that small angle approximation can be taken under consideration. For small angles in  $\triangle NOC$ ,  $i$  is the exterior angle.

By exterior angle theorem;

$$\therefore i = \angle NOM + \angle NCM$$

$$i = \frac{MN}{OM} + \frac{MN}{MC} \dots (i)$$

Similarly  $r = \angle NCM - \angle NIM$

$$= \frac{MN}{MC} - \frac{MN}{MI} \dots (ii)$$

By Snell's law

$$n_1 \sin i = n_2 \sin r$$

for small angles

$$n_1 i = n_2 r$$

substituting  $i$  and  $r$  from (i) and (ii) we get

$$\frac{n_1}{OM} + \frac{n_2}{MI} = \frac{n_2 - n_1}{MC}$$

Applying Cartesian coordinates

$OM = -u$ ,  $MI = +v$ ,  $MC = +R$

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$ii. \frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$R = -6$  cm,  $u = -3$  cm,  $n_1 = 1.5$   $n_2 = 1$

$$\frac{1}{v} + \frac{1.5}{3} = \frac{1-1.5}{-6}$$

$$\frac{1}{v} = \frac{0.5}{6} - \frac{1.5}{3}$$

$$\frac{1}{v} = \frac{0.5-3}{6}$$

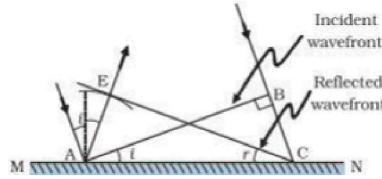
$$\frac{1}{v} = \frac{-2.5}{6}$$

$$v = -2.4 \text{ cm}$$

from the left surface inside the sphere

OR

i. **Huygen's principle** Each point of the wavefront is the source of a secondary disturbance and the wavelets emanating from these points spread out in all directions with the speed of the wave. These wavelets emanating from the wavefront are usually referred to as secondary wavelets, a common tangent to all these spheres gives the new position of the wavefront at a later time.



#### Verification of law of reflection

In  $\triangle AEC$  &  $\triangle CBA$

$$EC = AB \text{ (c x t each)}$$

$$\angle AEC = \angle CBA (90^\circ \text{ each})$$

$$AC = AC \text{ (common side)}$$

By RHS congruency  $\triangle AEC \cong \triangle CBA$

$$\Rightarrow \angle i = \angle r$$

Hence the law of reflection is verified.

ii.  $m = +3$ ,  $f = -12$  cm,  $u = ?$

$$m = -\frac{v}{u} = 3 \Rightarrow v = -3u$$

using mirror formula

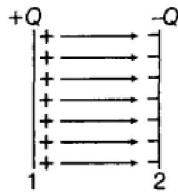
$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{-3u} + \frac{1}{u} = \frac{1}{-12}$$

$$u = -8 \text{ cm}$$

Hence the distance of the object from the mirror is 8 cm

32. i. Let the total charge on the plates of the below capacitor is  $+Q$  and  $-Q$  respectively.



$\therefore$  The potential difference between the plates of the above capacitor of capacitance  $C$  for an infinitesimal charge  $q$  is  $q/C$ .

$$\therefore \text{Potential of condenser} = q/C$$

Small amount of work done in giving an additional charge  $dq$  to the condenser,

$$dW = \frac{q}{C} \times dq$$

$\therefore$  Total work done in giving a charge  $Q$  to the condenser,

$$W = \int_{q=0}^{q=Q} \frac{q}{C} dq = \frac{1}{C} \left[ \frac{q^2}{2} \right]_{q=0}^{q=Q} \Rightarrow W = \frac{1}{C} \frac{Q^2}{2}$$

As, an electrostatic force is conservative, this work is stored in the form of potential energy ( $U$ ) of the condenser.

$$U = W = \frac{1}{2} \frac{Q^2}{C}$$

$$\because Q = CV \Rightarrow U = \frac{1}{2} \frac{(CV)^2}{C} = \frac{1}{2} CV^2$$

$$\because CV = Q \Rightarrow U = \frac{1}{2} QV$$

$$\text{Hence, } U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$$

Energy density ( $u$ ) is defined as the total energy per unit volume of the condenser.

$$\text{i.e., } u = \frac{\text{Total energy (U)}}{\text{Volume (V)}} = \frac{\frac{1}{2} CV^2}{Ad}$$

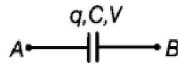
Using,  $C = \frac{\epsilon_0 A}{d}$  and  $V = Ed$  (Where  $V$  is the potential difference and  $E$  is the Electric field existing between the plates)

$$\text{We get, } u = \frac{1}{2} \left( \frac{\epsilon_0 A}{d} \right) \left( \frac{E^2 d^2}{Ad} \right) = \frac{1}{2} \epsilon_0 E^2$$

Here, Energy density between plates of capacitors is directly proportional to electric field that exists between the plates of capacitor.

ii. Initial condition :

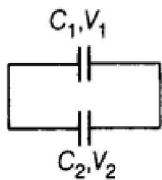
If we consider a charged capacitor of capacitance  $C$  with potential difference  $V$ , then its charge would be given,  $q = CV$



and energy stored in it is given by

$$U_1 = \frac{1}{2} CV^2 \dots \text{(i)}$$

When this charged capacitor is connected to uncharged capacitor,



Let the common potential be  $V_1$ , the charge flow from first capacitor to the other capacitor unless both the capacitor attains the common potential.

$$\Rightarrow Q_1 = CV_1 \text{ and } Q_2 = CV_2$$

Applying conservation of charge,

$$Q = Q_1 + Q_2 \Rightarrow CV = CV_1 + CV_2$$

$$\Rightarrow V = V_1 + V_2 \Rightarrow V_1 = \frac{V}{2} \text{ [hence voltage will be equally divided between the capacitors]}$$

Total energy stored in both the capacitor is

$$U_2 = \frac{1}{2} CV_1^2 + \frac{1}{2} CV_2^2 \Rightarrow U_2 = \frac{1}{2} C \left( \frac{V}{2} \right)^2 + \frac{1}{2} C \left( \frac{V}{2} \right)^2$$

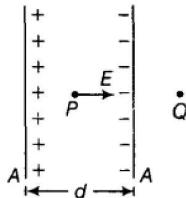
$$U_2 = \frac{2CV^2}{8} = \frac{1}{4} CV^2$$

From Eqs. (i) and (ii), we get,  $U_2 < U_1$

It means that energy stored in the combination is less than that stored initially in the single capacitor. It is due to the fact that when the charge is transferred from one capacitor to another capacitor energy is wasted in transferring the charge.

OR

a. Consider the figure shown below:



i. Electric field due to the plate of the positive charge of charge density  $+\sigma$  at point P, is given by

$$E_1 = \sigma/2\epsilon_0$$

Magnitude of electric field due to the other plate of negative charge density  $-\sigma$ , is given by

$$E_2 = -\sigma/2\epsilon_0$$

In , the inner region between the plates 1 and 2 , electric field due to the two charged plates add up, is given by

$$E_{\text{net}} = E_1 + E_2 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

Outside the plate, electric field will be equal to zero because of the opposite directions of the electric fields  $E_1$  and  $E_2$  there.

ii. Potential difference between the plates of the capacitor is given by

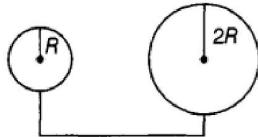
$$V = Ed = \sigma d / \epsilon_0 \quad (\because E = \sigma / \epsilon_0)$$

iii. Capacitance of the capacitor is given by

$$(\because Q = CV)$$

$$C = \frac{Q}{V} = \frac{\sigma A}{\sigma d} \epsilon_0 = \frac{\epsilon_0 A}{d}$$

b. Consider the figure shown below:



Potential at the surface of the sphere of radius R,

$$\begin{aligned} \{tex\} &= \frac{kq}{R} \quad [\because q = \sigma \times 4\pi R^2] \\ &= \frac{k\sigma 4\pi R^2}{R} = \sigma k 4\pi R = 4k\sigma\pi R \end{aligned}$$

Potential at the surface of the second sphere of radius twice the previous one i.e.  $2R$ ,

$$\begin{aligned} &= \frac{kq}{2R} \quad [\because q = \sigma \times 4\pi(2R)^2 = 16\sigma\pi R^2] \\ &= \frac{k16\sigma\pi R^2}{2R} = 8k\sigma\pi R \end{aligned}$$

We know that charge always flows from the higher potential surface to lower potential surface. Since the potential of the bigger sphere is more, so charge will flow from sphere of radius  $2R$  to the sphere of radius  $R$  after connecting both the spheres by a conducting wire

33. Inductance,  $L = 80 \text{ mH} = 80 \times 10^{-3} \text{ H}$

Capacitance,  $C = 60 \mu\text{F} = 60 \times 10^{-6} \text{ F}$

Supply voltage,  $V = 230 \text{ V}$

Frequency,  $\nu = 50 \text{ Hz}$

Angular frequency,  $\omega = 2\pi\nu = 100\pi \text{ rad/s}$

Peak voltage,  $V_0 = V\sqrt{2} = 230\sqrt{2} \text{ V}$

a. Maximum current is given as:

$$\begin{aligned} I_0 &= \frac{V_0}{(\omega L - \frac{1}{\omega C})} \\ &= \frac{230\sqrt{2}}{\left(100\pi \times 80 \times 10^{-3} - \frac{1}{100\pi \times 60 \times 10^{-6}}\right)} \\ &= \frac{230\sqrt{2}}{\left(8\pi - \frac{1000}{6\pi}\right)} = -11.63 \text{ A} \end{aligned}$$

The negative sign appears because  $\omega L < \frac{1}{\omega C}$

Amplitude of maximum current,  $|I_0| = 11.63\text{A}$

Hence, rms value of current.  $I = \frac{I_0}{\sqrt{2}} = \frac{11.63}{\sqrt{2}} = 8.22\text{ A}$

b. Potential difference across the inductor.

$$v_L = I \times \omega L$$

$$= 8.22 \times 100 \pi \times 80 \times 10^{-3}$$

$$= 206.61\text{ V}$$

Potential difference across the capacitor,

$$V_c = I \times \frac{1}{\omega C}$$

$$= 8.22 \times \frac{1}{100\pi \times 60 \times 10^{-6}} = 436.84\text{V}$$

c. Average power consumed over a **complete cycle by the source** to the inductor is zero as actual voltage leads the current by  $\frac{\pi}{2}$ .

d. Average power consumed over a complete cycle by the source to the capacitor is zero as voltage lags current by  $\frac{\pi}{2}$ .

e. The total power absorbed (averaged over one cycle) is zero.

OR

According to the above figure, the total current 'i' is divided into two parts  $i_2$  through R and  $i_1$  through a series combination of C and L.

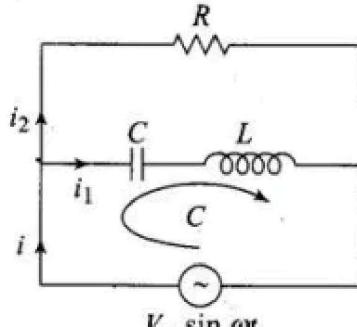
So, we get  $i = i_1 + i_2$

As,  $V_m \sin \omega t = R i_2$ , [from the circuit diagram, we find that]

$$\Rightarrow i_2 = \frac{V_m \sin \omega t}{R} \dots \text{(i)}$$

Let  $q_1$  is charge on the capacitor at any time t,  $i_1$  is the current in the lower circuit.

thus, by Applying KVL in the lower circuit as shown in the figure,



$$V_m \sin \omega t - \frac{q_1}{C} - \frac{L di_1}{dt} = 0$$

$$\Rightarrow \frac{q_1}{C} + \frac{L d^2 q_1}{dt^2} = V_m \sin \omega t \dots \text{(ii)}$$

$$\text{Let } q_1 = q_m \sin(\omega t + \phi) \dots \text{(iii)}$$

$$\therefore i_1 = \frac{dq_1}{dt} = q_m \omega \cos(\omega t + \phi)$$

$$\Rightarrow \frac{d(i_1)}{dt} = \frac{d^2 q_1}{dt^2} = -q_m \omega^2 \sin(\omega t + \phi)$$

Now putting these values in eq. (ii), we get

$$q_m \left[ \frac{1}{C} + L(-\omega^2) \right] \sin(\omega t + \phi) = V_m \sin \omega t$$

If  $\phi = 0$  and  $\left( \frac{1}{C} - L\omega^2 \right) > 0$

$$\text{then } q_m = \frac{V_m}{\left( \frac{1}{C} - L\omega^2 \right)} \dots \text{(iv)}$$

From Eq. (iii),  $i_1 = \frac{dq_1}{dt} = \omega q_m \cos(\omega t + \phi)$

$$\text{Using eq. (iv), } i_1 = \frac{\omega V_m \cos(\omega t + \phi)}{\frac{1}{C} - L\omega^2}$$

$$\text{Taking } \phi = 0; i_1 = \frac{V_m \cos(\omega t)}{\left( \frac{1}{\omega C} - L\omega \right)} \dots \text{(v)}$$

From Eqs. (i) and (v), we find that  $i_1$  and  $i_2$  are out of phase by  $\frac{\pi}{2}$

$$\text{Now, } i_2 + i_1 = \frac{V_m \sin \omega t}{R} + \frac{V_m \cos \omega t}{\left( \frac{1}{\omega C} - L\omega \right)}$$

Let  $\frac{V_m}{R} = A \cos \phi$  and  $\frac{V_m}{\left(\frac{1}{\omega C} - L\omega\right)} = A \sin \phi$

$$\therefore i_1 + i_2 = A \cos \phi \sin \omega t + A \sin \phi \cos \omega t$$

$$= A \sin(\omega t + \phi)$$

Where  $A = \sqrt{(A \cos \phi)^2 + (A \sin \phi)^2}$

$$\text{and } \phi = \tan^{-1} \frac{B}{A} C = \left[ \frac{V_m^2}{R^2} + \frac{V_m^2}{\left(\frac{1}{\omega C} - L\omega\right)^2} \right]^{1/2}$$

$$\text{and } \phi = \tan^{-1} \frac{R}{\left(\frac{1}{\omega C} - L\omega\right)}$$

$$\text{Hence, } i = i_1 + i_2 = \left[ \frac{V_m^2}{R^2} + \frac{V_m^2}{\left(\frac{1}{\omega C} - L\omega\right)^2} \right]^{1/2} \sin(\omega t + \phi)$$

$$\text{or } \frac{i}{V_m} = \frac{1}{Z} = \left[ \frac{1}{R^2} + \frac{1}{\left(\frac{1}{\omega C} - L\omega\right)^2} \right]^{1/2}$$

This is the expression for impedance  $Z$  of the circuit. Hence these are the required results.