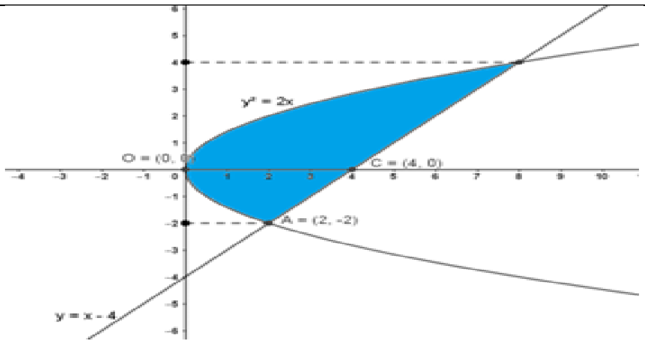
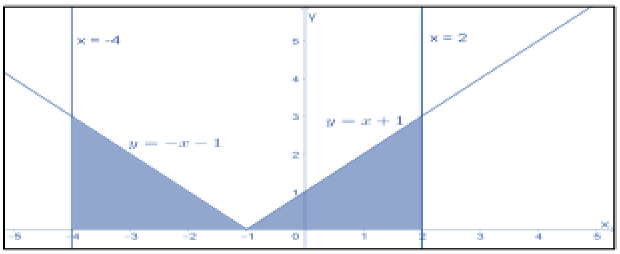
ST. JOSEPH PUBLIC SCHOOL

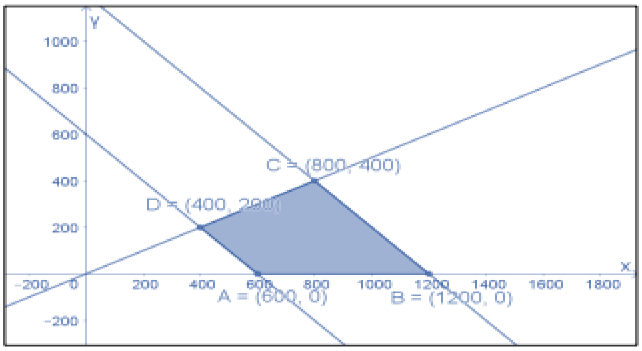
Kota Barrage Road, Kota-6 (Raj.)

C.B.S.E. New Delhi, **MATHEMATICS**	**SOLUTIONS**
Class: **XII**	**2025-26**
MM: **80**	

	SECTION-A (MCQs of 1 mark each)	
S.NO.	**Hints / Solutions**	**MARKS**
1	C	1
2	A	1
3	C	1
4	B	1
5	B	1
6	A	1
7	D	1
8	A	1
9	B	1
10	C	1
11	B	1
12	B	1
13	C	1
14	B	1
15	C	1
16	C	1
17	B	1
18	B	1
19	C	1
20	C	1
	SECTION B (VSA type questions of 2 marks each)	
21	$\tan\left(\tan^{-1}(-1)+\frac{\pi}{3}\right)=\tan\left(-\frac{\pi}{4}+\frac{\pi}{3}\right)$ $=\frac{\tan\frac{\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{4}}$ $=\frac{\sqrt{3}-1}{1+\sqrt{3}}$ *or* $2-\sqrt{3}$ **OR** For domain, $-1\leq 3x-2\leq 1$ $\Rightarrow 1\leq 3x\leq 3$ $\Rightarrow \frac{1}{3}\leq x\leq 1$ So, domain of $\cos^{-1}(3x-2)$ is $\left[\frac{1}{3},1\right]$	½ 1 ½ ½ ½ ½ ½
22	$y=x^{1/x}$ $\log y=\frac{1}{x}\log x$ $\frac{1}{y}\frac{dy}{dx}=-\frac{\log x}{x^2}+\frac{1}{x^2}\ \Rightarrow \frac{dy}{dx}=x^{\frac{1}{x}}\ \frac{(1-\log x)}{x^2}$ $\left(\frac{dy}{dx}\right)_{x=1}=1$	 ½ 1 1/2
23	Given definite integral $=\int_0^{\frac{\pi}{4}}\sqrt{(sinx+cosx)^2}dx$ $=\int_0^{\frac{\pi}{4}}(sinx+cosx)dx$ $=[-cosx+sinx]_0^{\frac{\pi}{4}}$ $=1$ **OR** For correct fig.	1 ½ ½ ½ 1 ½

	Required area $= \int_2^4 \frac{1}{2}\ (8-y)dy$ $= \frac{1}{2}\left\|8y - \frac{y^2}{2}\right\|_2^4$ $= 5$	
24	$f'(5) = \lim_{h\to 0}\frac{f(5+h)-f(5)}{h} = \lim_{h\to 0}\frac{f(5)f(h)-f(5)}{h}$ $= \lim_{h\to 0}\frac{2f(h)-2}{h}\quad [\because f(5)=2]$ $= 2\lim_{h\to 0}\frac{f(h)-1}{h} = 2\lim_{h\to 0}\frac{f(h)-f(0)}{h} = 2f'(0)$ $= 2(3)\quad [\because f'(0)=3]$ $= 6$	½ 1 ½
25	$\cos\frac{\pi}{4} = \frac{\|\alpha\,.\,1+0+\beta\|}{\sqrt{\alpha^2+\beta^2+25}\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2}} = \frac{\|\alpha+\beta\|}{\sqrt{\alpha^2+\beta^2+25}\sqrt{2}}$ Squaring both sides, we get $\alpha^2+\beta^2+2\alpha\beta = \alpha^2+\beta^2+25$ $\Rightarrow \alpha\beta = \frac{25}{2}$	1 ½ ½
	SECTION-C	
26	Let 'a' be the side of the triangle, so $\frac{da}{dt} = 3$ cm/s Now area of an equilateral triangle, $A = \frac{\sqrt{3}}{4}a^2$ $\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}a}{2}\times\frac{da}{dt}$ $\therefore \left.\frac{dA}{dt}\right]_{a=15\text{cm}} = \frac{\sqrt{3}\times 15}{2}\times 3 = \frac{45\sqrt{3}}{2}\text{cm}^2/\text{s}$	½ 1 ½ 1/2
27	$x^y = e^{x-y}$ Taking log of both sides $y\log x = (x-y)\log e$ $y\ \log x + y = x$ (since $\log e = 1$) $\Rightarrow\ y = \frac{x}{1+\log x}$ Differentiating with respect to x $\frac{dy}{dx} = \frac{(1+\log x).1 - x\frac{1}{x}}{(1+\log x)^2}$ $= \frac{\log x}{(\log e+\log x)^2}$ $= \frac{\log x}{(\log(x\,e))^2}$ Now $\left.\frac{dy}{dx}\right]_{x=e} = \frac{\log e}{(\log\ e^2)^2} = \frac{1}{(2\log e)^2} = \frac{1}{2^2} = \frac{1}{4}$ (as $\log e = 1$) **OR** $\frac{dx}{d\theta} = a(1-\cos\theta)$, $\frac{dy}{d\theta} = a(0+\sin\theta)$, $\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1-\cos\theta)}$ $= \frac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2\sin^2\left(\frac{\theta}{2}\right)} = \cot\frac{\theta}{2}$ $\Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{2}cosec^2\left(\frac{\theta}{2}\right)\frac{d\theta}{dx}$ $= -\frac{1}{2a}cosec^2\left(\frac{\theta}{2}\right)\frac{1}{2\sin^2\left(\frac{\theta}{2}\right)}$ $= -\frac{1}{4a}cosec^4\left(\frac{\theta}{2}\right)$	1 1 1 1 1 1

28		1
	Solving $y^2 = 2x$ and $y = x - 4$, we get	
	$y = 4$ or -2	
	Required area $= \int_{-2}^{4}\left[(y + 4) - \frac{y^2}{2}\right] dy$	1 ½
	$= \left\| \frac{y^2}{2} + 4y - \frac{1}{6}y^3 \right\|_{-2}^{4}$	½
	$= 18$	
	OR	
		1
	$\int_{-4}^{2}\|x + 1\|dx = \int_{-4}^{-1}(-x - 1)\, dx + \int_{-1}^{2}(x + 1)\, dx$	½
	$= -\left.\frac{(x+1)^2}{2}\right]_{-4}^{-1} + \left.\frac{(x+1)^2}{2}\right]_{-1}^{2}$	½
	$= -\left(0 - \frac{9}{2}\right) + \left(\frac{9}{2} - 0\right) = 9$	½
	It represent the area of shaded region bounded by the curve $y = \|x + 1\|$, $x -$ axis and the lines $x = -4$ *and* $x = 2$	½
29	**Rewriting the lines, we get**	½
	$\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})$ **and** $\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} + 2\hat{j} - 2\hat{k})$	
	Let $\vec{a}_1 = \hat{i} - 2\hat{j} + 3\hat{k},\ \vec{a}_2 = \hat{i} - \hat{j} - \hat{k},\ \vec{b}_1 = -\hat{i} + \hat{j} - 2\hat{k},\ \vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$	
	Note that the dr'sof given lines are not proportional so, they are not parallel lines.	
	The lines will be skew if they do not intersect each other also.	
	Here $\vec{a}_2 - \vec{a}_1 = \hat{j} - 4\hat{k},\ \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = 2\hat{i} - 4\hat{j} - 3\hat{k}$	
	Consider $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)$	1
	$= (\hat{j} - 4\hat{k}).(2\hat{i} - 4\hat{j} - 3\hat{k}) = 8 \neq 0$	
	Hence lines will not intersect. So the lines are skew.	½
	Shortest Distance $= \frac{\|(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)\|}{\|\vec{b}_1 \times \vec{b}_2\|}$	
	$= \frac{8}{\sqrt{4 + 16 + 9}} = \frac{8}{\sqrt{29}}$	1
	OR	
	The line through $(2, -1, 3)$ parallel to the z-axis is given by	
	$\vec{r} = (2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + \lambda(\hat{k})$	1
	Any point on this line is P $(2, -1, 3 + \lambda)$	½
	Any point on the given line $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \mu(3\hat{\imath} + 6\hat{\jmath} + 2\hat{k})$ is	½
	Q $(2 + 3\mu, -1 + 6\mu, 2 + 2\mu)$	
	For the intersection point	½
	Q $(2 + 3\mu, -1 + 6\mu, 2 + 2\mu)$ = P $(2, -1, 3 + \lambda) \Rightarrow 2 = 2 + 3\mu \quad \Rightarrow \mu = 0$	
	The point of intersection is $(2, -1, 2)$	½
	The distance from $(2, -1, 3)$ to $(2, -1, 2)$ is clearly 1 unit.	

30	Sketching the graph  Corner points A(600,0), B(1200,0), C(800,400), D(400,200) Values of Z: $Z_A = 1200, Z_B = 2400, Z_C = 2000, Z_D = 1000$ Maximum $Z = 2400$ when $x = 1200$ and $y = 0$	1 ½ ½ ½ ½
31	**Let E_1 : The applicant is a male** **E_2 : The applicant is a female** **A: The candidate chosen will have distinction in the written test.** $P(E_1)=\frac{1}{3}, P(E_2)=\frac{2}{3}, P(A\mid E_1)=0.4, P(A\mid E_2)=0.35$ $\therefore P(A)=P(E_1)P(A\mid E_1)+P(E_2)P(A\mid E_2)$ $=\frac{1}{3}\times 0.4+\frac{2}{3}\times 0.35$ $=\frac{11}{30}$	½ 1 1 ½
	SECTION-D	
32	$AB = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ -2 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$ So, $A^{-1} = B$ and $B^{-1} = A$ Given system of equations is $3x - 6y - z = 3,\ \ 2x - 5y - z + 2 = 0,\ \ -2x + 4y + z = 5$ In matrix form it can be written as: $AX = C$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $C = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}$ Here $\lvert A\rvert = -3 - 0 + 2 = -1 \neq 0$ So, the system is consistent and has unique solution given by the expression $X = A^{-1}C = BC$ $\Rightarrow X = \begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 21 \end{bmatrix}$ Thus $x = 2, y = -3, z = 21$	1 ½ ½ ½ ½ ½ 1 ½

33	$I=\int \frac{\cos x}{(4+\sin^2 x)(5-4\cos^2 x)}dx$	
	$=\int \frac{\cos x}{(4+\sin^2 x)(1+4\sin^2 x)}dx$	1
	$\sin x = t$ gives	½
	$I=\int \frac{dt}{(4+t^2)(1+4t^2)}$	
	$=-\frac{1}{15}\int \frac{dt}{4+t^2}+\frac{4}{15}\int \frac{dt}{1+4t^2}$ $(\because \text{using Partial Fraction})$	2
	$=-\frac{1}{30}\tan^{-1}\left(\frac{t}{2}\right)+\frac{2}{15}\tan^{-1}(2t)+C$	1
	$=-\frac{1}{30}\tan^{-1}\left(\frac{\sin x}{2}\right)+\frac{2}{15}\tan^{-1}(2\sin x)+C$	½
	OR	
	$I=\int_0^{\pi} \frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$	
	$=2\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$	½
	$=2\int_0^{\pi/2} \frac{\sec^2 x}{a^2+b^2\tan^2 x}dx$	1
	$\tan x = t$ gives	
	$I=2\int_0^{\infty} \frac{dt}{a^2+b^2t^2}$	½
	$=\frac{2}{b^2}.\frac{b}{a}\tan^{-1}\left(\frac{bt}{a}\right)\Big]_0^{\infty}$	½ 1 ½
	$=\frac{\pi}{ab}$	1
34	$y+\frac{d}{dx}(xy) = x(\sin x + x)$	
	$\Longrightarrow y + (x\frac{dy}{dx} + y) = x(\sin x + x)$	1
	$\Longrightarrow 2y + x\frac{dy}{dx} = x(\sin x + x)$	
	$\Longrightarrow \frac{dy}{dx} + \frac{2y}{x} = (\sin x + x)$	
	This a linear differential equation of the form $\frac{dy}{dx} + Py = Q$	1
	P= $\frac{2}{x}$,Q=$(\sin x + x)$	
	I.F $=e^{\int \frac{2}{x}dx} = e^{2\log x} = e^{\log x^2} = x^2$	1
	Solution will be y . I.F $=\int Q.IF\,dx$	
	$yx^2 = \int(\sin x + x)x^2$ dx	
	$yx^2 = \int \sin x.x^2\,dx + \int x^3\,dx$	1
	$\Longrightarrow yx^2 = -x^2\cos x + 2\int x\cos x\,dx + \frac{x^4}{4}$ + C	
	$\Longrightarrow yx^2 = -x^2\cos x$ +2 $(x\sin x + \cos x) + \frac{x^4}{4}$ + C	1
	Which is the required solution **OR**	
	$2y\,e^{x/y}dx + (y - 2x\,e^{x/y})\,dy = 0$	
	$\Longrightarrow \frac{dx}{dy} = \frac{2x\,e^{x/y}-y}{2y\,e^{x/y}} = \frac{2\frac{x}{y}e^{\frac{x}{y}}-1}{2\,e^{\frac{x}{y}}}$	1
	It is a homogeneous differential equation.	
	Let $x = vy \Longrightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$	1
	$v + y\frac{dv}{dy} = \frac{2ve^v-1}{2e^v}$	
	$\Longrightarrow y\frac{dv}{dy} = \frac{2ve^v-1}{2e^v} - v = \frac{2ve^v-1-2ve^v}{2e^v}$	1
	$\Longrightarrow y\frac{dv}{dy} = \frac{-1}{2e^v}$	
	$\Longrightarrow 2e^v\,dv = -\frac{dy}{y}$	
	$\int 2e^v\,dv = -\int \frac{dy}{y}$	
	$\Longrightarrow 2e^v = -\log\|y\| + C$	1
	$\Longrightarrow 2e^{\frac{x}{y}}$ +" $\log\|y\|$ = C	
	When $x = 0$, $y = 1$, C = 2	
	Required solution 2 $e^{\frac{x}{y}}$ + log$\|y\|$ = 2	1

35	(a)As lines are intersecting, $(\overrightarrow{a_2}-\overrightarrow{a_1}).((\overrightarrow{b_1}\times\overrightarrow{b_2})=0$ $\Rightarrow \begin{vmatrix} 3 & 1-b & -3 \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}=0$ $\Rightarrow b=2$ Any point on the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ is $(2\lambda+1, 3\lambda+2, 4\lambda+3),\ \lambda\in R$ For the point of intersection, this point must lie on the line $\frac{x-4}{5}=\frac{y-1}{2}=z$ $\Rightarrow \frac{2\lambda+1-4}{5}=\frac{3\lambda+2-1}{2}=4\lambda+3$ $\Rightarrow \lambda=-1$ $\therefore$ point of intersection is $(-1,-1,-1)$	1 1 1 1 ½ ½
36	I. Traffic flow is not reflexive as $(A,A)\notin R$ (or no major spot is connected with itself) II. Traffic flow is not transitive as $(A,B)\in R$ and $(B,E)\in R$, but $(A,E)\notin R$ III A. $R=\{(A,B),(A,C),(A,D),(B,C),(B,E),(C,E),(D,E),(D,C)\}$ Domain $=\{A,B,C,D\}$ Range $=\{B,C,D,E\}$ **OR** III B. No, the traffic flow doesn't represent a function as A has three images.	1 1 1 ½+ ½ 1+1
37	**(i)** $V=x^2y \Rightarrow y=\frac{V}{x^2}$ ……………(i) Hence, $S=2x^2+4xy=2x^2+\frac{4V}{x}$ **(ii)** $\frac{dS}{dx}=4\left(x-\frac{V}{x^2}\right)$ **(iii)** (a) $\frac{dS}{dx}=0 \Rightarrow V=x^3 \Rightarrow x^2y=x^3 \Rightarrow y=x$ $\frac{d^2S}{dx^2}=4\left(1+\frac{2V}{x^3}\right)>0 \Rightarrow$ S is minimum if $y=x$. **OR** **(iii)** (b) $V=\frac{1}{4}(Sx-2x^3) \Rightarrow \frac{dV}{dx}=\frac{1}{4}(S-6x^2)$	1 1 1 1 1+1
38	**Let A: Event that chosen seed germinates.** **B: Event that Brinjal seed is chosen.** **C: Event that Cabbage seed is chosen.** **R: Event that Radish seed is chosen.** $P(B)=\frac{10}{30};\ P(C)=\frac{12}{30};\ P(R)=\frac{8}{30};$ $P\left(\frac{A}{B}\right)=\frac{25}{100};\ P\left(\frac{A}{C}\right)=\frac{35}{100};\ P\left(\frac{A}{R}\right)=\frac{40}{100}$ **(i)** $P(A)=P(B).P\left(\frac{A}{B}\right)+P(C).P\left(\frac{A}{C}\right)+P(R).P\left(\frac{A}{R}\right)$ $=\frac{10}{30}\times\frac{25}{100}+\frac{12}{30}\times\frac{35}{100}+\frac{8}{30}\times\frac{40}{100}$ $=\frac{990}{3000}$ or $\frac{33}{100}$ **(ii)** (a) $P\left(\frac{C}{A}\right)=\frac{P(C).P\left(\frac{A}{C}\right)}{P(B).P\left(\frac{A}{B}\right)+P(C).P\left(\frac{A}{C}\right)+P(R).P\left(\frac{A}{R}\right)}$ $=\frac{\frac{12}{30}\times\frac{35}{100}}{\frac{990}{3000}}$ $=\frac{42}{99}$ or $\frac{14}{33}$	1 ½ ½ 1 1